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Abstract
We investigate the quantum effects of the nonlocal gauge invariant operator $\frac{1}{D^2} F_{\mu\nu} * \frac{1}{D^2} F^{\mu\nu}$ in the noncommutative $U(1)$ action and its consequences to the infrared sector of the theory. Nonlocal operators of such kind were proposed to solve the infrared problem of the noncommutative gauge theories evading the questions on the explicit breaking of the Lorentz invariance. More recently, a first step in the localization of this operator was accomplished by means of the introduction of an extra tensorial matter field, and the first loop analysis was carried out (Blaschke *et al* (2009 *Eur. Phys. J.* C **62** 433–43)). We will complete this localization avoiding the introduction of new degrees of freedom beyond those of the original action by using only BRST doublets. This will allow us to conduct a complete BRST algebraic study of the renormalizability of the theory, following Zwanziger's method of localization of nonlocal operators in QFT.

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1. Introduction

The year 1999 witnessed two major developments in the noncommutative quantum field theory program. In the first one, Seiberg and Witten [1], inspired by the previously known result that the low energy limit of open strings could lead both to a gauge theory defined on a noncommutative space as well as to a usual commutative gauge theory, depending only on gauge choices, announced the existence of what became called the Seiberg–Witten map

between noncommutative and commutative gauge theories. This achievement was then fully tested and confirmed by several authors in both the general structures of gauge transformations as in specific examples of gauge theories (we make a short list of references which is far from being complete [2–11]). It also opened a window to an alternative approach to the quantum properties of the noncommutative theories.

The second development just revealed the kind of difficulties one has to face when tackling the renormalization of field theories in the noncommutative space. An intrinsic mixing between high and low energy scales was associated with the noncommutativity of spacetime, generating divergences which in the general case make these theories nonrenormalizable as they stand [12]; the case of noncommutative gauge theories being no exception [13]. Recently, it was finally understood that this infrared/ultraviolet (IR/UV) mix is still present even after a Seiberg–Witten map [14], showing that the commutative theories generated by their noncommutative counterparts suffer from the same nonrenormalizability.

It took some time until the first proposal appeared in order to cure a noncommutative scalar theory from this IR divergence [15]. The basic idea was to alter the free propagator of the theory through the introduction of an harmonic potential, then changing its low energy behavior. This in fact made the theory convergent in the infrared region, but at the cost of explicitly breaking translation invariance. In [16] this problem was circumvented by the introduction of a nonlocal term, again assuring that the IR/UV mixing would be cured for a scalar theory. Soon, this proposal was generalized to the case of a noncommutative gauge theory [17]. The main idea was still the same, to change the low energy pattern of the theory, and this was obtained through the introduction of a nonlocal term one more time. The practical effect of this term is a modified free propagator of the gauge field, which acquires a $1/k^4$ pole, consistently defined in Euclidean spacetime. This is how the infrared regime of the theory gets modified. Again, we still have a problem with this approach in the way it was presented up to this point, as the nonlocality is not adequate to match the requisites of the quantum action principle (QAP) [18] here taken as valid in the noncommutative space (even though the quantun action principle has no proof of validity in the noncommutative environment, its use became standard after the results of [19–22]; we will have more to say about this in section 4.1). The way out would be to find an equivalent local action meeting the same properties of the previous one. So the quantum study of such theory was awaited until more recently when a way to localize this nonlocal action was found. Then a one-loop analysis was finally carried out [23]. This was an important achievement, but once more there is an undesirable feature: the introduction of an extra field in the theory, creating extra degrees of freedom not present in the original noncommutative gauge theory. A natural question would be to ask if this is an unavoidable price to be paid in order to have a possibly renormalizable noncommutative theory with gauge interactions.

Our intention here will be focused on presenting an alternative scenario of localization, paving the way to a renormalizable noncommutative gauge field theory, but avoiding to introduce any extra degree of freedom.

In section 2, we present the nonlocal action, its localization via doublet fields and the resulting BRST symmetry. In section 3, the equations compatible with the quantum action principle are derived. Section 4 is dedicated to the analysis of the quantum stability of the theory. In this section we pay special attention to possible UV quantum corrections that can spoil the IR renormalizability of the two-point function. The definitive form of the propagator is finally obtained, showing a modification from the classical starting one. In section 5, we show our conclusion.

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2. BRST in Euclidean space

The nonlocal action that we will study is

$$S_{\mathrm{NL}} = \int \mathrm{d}^4 x \left\{ \frac{1}{4} F_{\mu\nu} * F^{\mu\nu} + \gamma^4 \frac{1}{4} \frac{1}{D^2} F_{\mu\nu} * \frac{1}{D^2} F^{\mu\nu} \right\}. \tag{1}$$

We assume a Euclidian signature for the spacetime and an Abelian gauge group, with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}g [A_\mu \overset{*}{,} A_\nu], \qquad D_\mu = \partial_\mu + \mathrm{i}g[\overset{*}{,} A_\mu]. \tag{2}$$

The commutator of two coordinates is $[x^\mu, x^\nu] = \mathrm{i}\Theta^{\mu\nu}$, where

$$\Theta = \begin{pmatrix} 0 & \theta & 0 & 0 \\ -\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta \\ 0 & 0 & -\theta & 0 \end{pmatrix}$$

and θ is the noncommutativity parameter [16].

This action gives to the gauge field propagator a more adequate behavior in the infrared for the noncommutative space:

$$\langle A(k)_\mu A_\nu(-k) \rangle = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \frac{k^2}{k^4 + \gamma^4}. \tag{3}$$

As pointed out in [17, 23], the infrared behavior of this kind of propagator decouples the ultraviolet and infrared regimes, and, then, the action (1) is a good candidate to generate a coherent quantum gauge theory in noncommutative space, without the IR/UV mix.

The action S_{NL} can be localized introducing a set of auxiliary tensorial fields. We use two pairs of complex conjugated fields $\overline{B}_{\mu\nu}, B_{\mu\nu}; \overline{\chi}_{\mu\nu}, \chi_{\mu\nu}$. We will see that only with such a structure, one can hope to get rid of the unwanted extra degrees of freedom. Anyway, the action

$$S_{\mathrm{LO}} = S_0 + S_{\mathrm{break}},$$

$$S_0 = \int \mathrm{d}^4 x \left\{ \frac{1}{4} F_{\mu\nu} * F^{\mu\nu} + \overline{\chi}_{\mu\nu} * D^2 B^{\mu\nu} + \overline{B}_{\mu\nu} * D^2 \chi^{\mu\nu} + \gamma^2 \overline{\chi}_{\mu\nu} * \chi^{\mu\nu} \right\},$$

$$S_{\mathrm{break}} = \int \mathrm{d}^4 x \left\{ -\mathrm{i}\frac{\gamma}{2} B_{\mu\nu} * F^{\mu\nu} + \mathrm{i}\frac{\gamma}{2} \overline{B}_{\mu\nu} * F^{\mu\nu} \right\}, \tag{4}$$

although representing the nonlocal operator of (1) in a localized form, still presents the problem that new degrees of freedom are being introduced by the auxiliary fields. This makes the physics content of the theory described by (4) different from that of a noncommutative $U(1)$ theory.

This problem can be solved by associating a ghost for each tensorial field introduced, in a way that a BRST structure of quartets will appear. This possibility of eliminating the extra degrees is the main reason for our choice of localization, as other attempts fail at this point. The action which attains this aim is

$$S_{\mathrm{LO}+G} = S_{0+G} + S_{\mathrm{break}}$$

$$S_{0+G} = \int \mathrm{d}^4 x \left\{ \frac{1}{4} F_{\mu\nu} * F^{\mu\nu} + \overline{\chi}_{\mu\nu} * D^2 B^{\mu\nu} + \overline{B}_{\mu\nu} * D^2 \chi^{\mu\nu} \right.$$

$$\left. + \gamma^2 \overline{\chi}_{\mu\nu} * \chi^{\mu\nu} - \overline{\psi}_{\mu\nu} * D^2 \xi^{\mu\nu} - \overline{\xi}_{\mu\nu} * D^2 \psi^{\mu\nu} - \gamma^2 \overline{\psi}_{\mu\nu} * \psi^{\mu\nu} \right\}. \tag{5}$$

The action S_{0+G} is left invariant by the set of BRST transformations:

$$sA_\mu = -D_\mu c, \qquad sc = -\frac{ig}{2}\{c \overset{*}{,} c\},$$

$$s\bar{c} = ib, \qquad sb = 0,$$

$$sF_{\mu\nu} = -ig[c \overset{*}{,} F_{\mu\nu}],$$

$$s\bar{\xi}_{\mu\nu} = \bar{B}_{\mu\nu} - ig\{c \overset{*}{,} \bar{\xi}_{\mu\nu}\}, \qquad s\bar{B}_{\mu\nu} = -ig[c \overset{*}{,} \bar{B}_{\mu\nu}], \qquad (6)$$

$$s\bar{\psi}_{\mu\nu} = \bar{\chi}_{\mu\nu} - ig\{c \overset{*}{,} \bar{\psi}_{\mu\nu}\}, \qquad s\bar{\chi}_{\mu\nu} = -ig[c \overset{*}{,} \bar{\chi}_{\mu\nu}],$$

$$sB_{\mu\nu} = \xi_{\mu\nu} - ig[c \overset{*}{,} B_{\mu\nu}], \qquad s\xi_{\mu\nu} = -ig\{c \overset{*}{,} \xi_{\mu\nu}\},$$

$$s\chi_{\mu\nu} = \psi_{\mu\nu} - ig[c \overset{*}{,} \chi_{\mu\nu}], \qquad s\psi_{\mu\nu} = -ig\{c \overset{*}{,} \psi_{\mu\nu}\},$$

where one can see the formation of a double quartet structure. This is an important point to highlight here: the structure of two quartets is essential for the localization process. A possible localization with only one quartet implies the use of the operator $D^2 D^2$ or other equivalent operator with four derivatives, and it is clear that this option leads to many nonrenormalizable vertices for the localizing fields. These vertices carry large momentum as expected by a theory that uses a field with canonical dimension 1 and ultraviolet dimension 0 [24]. In a commutative theory, this fact certainly destroys the renormalizability, but a deeper analysis is required in the case of noncommutative theories due to the structure of the UV/IR mix which is not very well known and the possibility of the softening of divergences [25]. For such reasons, we decided to use two quartets.

The action S_{0+G} can then be written as

$$S_{0+G} = \int d^4x \left\{ \frac{1}{4} F_{\mu\nu} * F^{\mu\nu} \right\} + s\Delta^{-1},$$

$$\Delta^{-1} = \int d^4x \{ \bar{\psi}_{\mu\nu} * D^2 B^{\mu\nu} + \bar{\xi}_{\mu\nu} * D^2 \chi^{\mu\nu} + \gamma^2 \bar{\psi}_{\mu\nu} \chi^{\mu\nu} \}. \qquad (7)$$

Once more we note that the physical degrees of freedom of the noncommutative $U(1)$ theory are being preserved. In our localized action (5), there is still a piece to be analyzed. The S_{break} sector of the action is not left invariant by the BRST transformations (6). This is the element that will bring a new physics to the pure $U(1)$ case. It is BRST transformed into

$$s S_{\text{break}} = \int d^4x \left\{ -i\frac{\gamma}{2} \xi_{\mu\nu} * F^{\mu\nu} \right\}. \qquad (8)$$

From this point on, we will assume that the Moyal product is rigid under quantum corrections. In the noncommutative space, the Moyal structure is intimately related to the gauge symmetry, and one cannot modify the first without damaging the latter. This can also be inferred from the fact that the only nontrivial cocycles of the BRST cohomology of gauge theories involve exclusively the terms constructed with the field strength and covariant derivatives at the level of the counterterms in the study of the quantum stability of the gauge action [18]. Naturally, in the noncommutative space, there is room for higher dimensional terms built explicitly with θ, field strengths and covariant derivatives, invariant and nontrivial in the BRST sense, which are not present at the original action. This is also seen by the method of consistent deformations of [26] applied to the present case of noncommutative deformations of Maxwell theory [6]. It is the Lorentz structure of the vertices of the theory together with gauge invariance which prohibits such counterterms. In [27], explicit calculations in noncommutative Chern–Simons theory showed these properties. Then, although the presence of $F_{\mu\nu}$ in (8) implies an infinite series of terms, the rigidity of the Moyal product determines that $F_{\mu\nu}$ is renormalized as a whole. This allows us to understand the breaking in (8) in a way analogous to that of a soft

breaking in a commutative theory (one can see that in zero θ order, this breaking is undoubtedly a soft breaking). The treatment of softly broken theories was recently formalized in [28]. We will need to study the renormalization of the theory together with the renormalization of the breaking itself. This is done by introducing a set of sources in a BRST doublet in such a way that the physical action is obtained when we set the sources to their physical values:

$$S_{\text{break}} = S_{\text{source}}|_{\text{phys}}$$

$$S_{\text{source}} = \int d^4x (\overline{J}_{\mu\nu\alpha\beta} * \{B^{\mu\nu} \overset{*}{,} F^{\alpha\beta}\} + J_{\mu\nu\alpha\beta} * \{\overline{B}^{\mu\nu} \overset{*}{,} F^{\alpha\beta}\} - \overline{Q}_{\mu\nu\alpha\beta} * \{\xi^{\mu\nu} \overset{*}{,} F^{\alpha\beta}\}), \tag{9}$$

where by $|_{\text{phys}}$ we mean that in this limit the sources attain their physical values,

$$J_{\mu\nu\alpha\beta} = \frac{i}{8}\gamma(\delta_{\mu\alpha}\delta_{\beta\nu} - \delta_{\mu\beta}\delta_{\alpha\nu}), \qquad \overline{J}_{\mu\nu\alpha\beta}| = -\frac{i}{8}\gamma(\delta_{\mu\alpha}\delta_{\beta\nu} - \delta_{\mu\beta}\delta_{\alpha\nu}),$$
$$Q_{\mu\nu\alpha\beta} = 0, \qquad \overline{Q}_{\mu\nu\alpha\beta}| = 0. \tag{10}$$

The BRST transformation of the sources,

$$s Q_{\mu\nu\alpha\beta} = J_{\mu\nu\alpha\beta} - ig\{c \overset{*}{,} Q_{\mu\nu\alpha\beta}\}, \qquad s J_{\mu\nu\alpha\beta} = -ig[c \overset{*}{,} J_{\mu\nu\alpha\beta}], \tag{11}$$

$$s \overline{Q}_{\mu\nu\alpha\beta} = \overline{J}_{\mu\nu\alpha\beta} - ig\{c \overset{*}{,} \overline{Q}_{\mu\nu\alpha\beta}\}, \qquad s \overline{J}_{\mu\nu\alpha\beta} = -ig[c \overset{*}{,} \overline{J}_{\mu\nu\alpha\beta}], \tag{12}$$

shows the doublet structure that we have already mentioned. The action (9) is now easily seen as an exact BRST variation, and the process altogether is a kind of an immersion of the original theory inside this more general one. Following this reasoning, we can now also rewrite the mass term $\gamma^2 \overline{\chi}_{\mu\nu}\chi_{\mu\nu} - \gamma^2 \overline{\psi}_{\mu\nu}\psi_{\mu\nu}$ in such a way that the mass parameter γ^2 only appears in the theory after taking the physical values for the sources $J, \overline{J}, Q, \overline{Q}$. This approach makes it easier to note that before this process, only the original degrees of freedom coming from the gauge field A_μ are present in the action.

The last steps needed for the BRST quantization are the definition of a gauge fixing, which we take as the noncommutative Landau gauge fixing,

$$S_{\text{gf}} = \int d^4x \{ib * \partial_\mu A^\mu + \overline{c} * \partial^\mu D_\mu c\}. \tag{13}$$

And finally, a set of Slavnov sources $\Omega, L, \overline{u}, u, \overline{v}, v, \overline{P}, P, \overline{R}, R, \overline{M}, M, \overline{N}, N$ are introduced in the action coupled to the nonlinear BRST transformations of the fields $A, c, \xi, \overline{\overline{\xi}}, B, \overline{B}, \psi, \overline{\psi}, \chi, \overline{\chi}$ and sources $Q, \overline{Q}, J, \overline{J}$ respectively.

The complete invariant action can then be written as

$$\Sigma = \int d^4x \left\{ \frac{1}{4} F_{\mu\nu} * F^{\mu\nu} + ib * \partial_\mu A^\mu + \overline{c} * \partial^\mu D_\mu c + \overline{\chi}_{\mu\nu} * D^2 B^{\mu\nu} + \overline{B}_{\mu\nu} * D^2 \chi^{\mu\nu} \right.$$

$$+ \overline{J}_{\mu\nu\alpha\beta} * \{B^{\mu\nu} \overset{*}{,} F^{\alpha\beta}\} + J_{\mu\nu\alpha\beta} * \{\overline{B}^{\mu\nu} \overset{*}{,} F^{\alpha\beta}\} - \overline{\psi}_{\mu\nu} * D^2 \xi^{\mu\nu} - \overline{\xi}_{\mu\nu} * D^2 \psi^{\mu\nu}$$

$$+ \frac{2}{3}\{\overline{J}_{\alpha\beta\sigma\lambda} \overset{*}{,} J^{\alpha\beta\sigma\lambda}\}\{\overline{\chi}_{\mu\nu} \overset{*}{,} \chi^{\mu\nu}\} - \frac{2}{3}\{\overline{J}_{\alpha\beta\sigma\lambda} \overset{*}{,} J^{\alpha\beta\sigma\lambda}\}[\overline{\psi}_{\mu\nu} \overset{*}{,} \psi^{\mu\nu}]$$

$$- \overline{Q}_{\mu\nu\alpha\beta} * \{\xi^{\mu\nu} \overset{*}{,} F^{\alpha\beta}\} - \Omega_\mu * D^\mu c - \frac{i}{2}L * g\{c \overset{*}{,} c\}$$

$$- i\overline{u}^{\mu\nu} * g\{c \overset{*}{,} \xi_{\mu\nu}\} + u^{\mu\nu} * (\overline{B}_{\mu\nu} - ig\{c \overset{*}{,} \overline{\xi}_{\mu\nu}\})$$

$$+ \overline{v}^{\mu\nu} * (\xi_{\mu\nu} - ig[c \overset{*}{,} B_{\mu\nu}]) - iv^{\mu\nu} * g\{c \overset{*}{,} \overline{B}_{\mu\nu}\}$$

$$- i\overline{P}^{\mu\nu} * g\{c \overset{*}{,} \psi_{\mu\nu}\} + P^{\mu\nu} * (\overline{\chi}_{\mu\nu} - ig\{c \overset{*}{,} \overline{\psi}_{\mu\nu}\})$$

$$+ \overline{R}^{\mu\nu} * (\psi_{\mu\nu} - ig[c \overset{*}{,} \chi_{\mu\nu}]) - iR^{\mu\nu} * g[c \overset{*}{,} \overline{\chi}_{\mu\nu}]$$

$$+\overline{M}^{\mu\nu\alpha\beta} * (J_{\mu\nu\alpha\beta} - \mathrm{i}g\{c \overset{*}{,} Q_{\mu\nu\alpha\beta}\}) + M^{\mu\nu\alpha\beta} * (\overline{J}_{\mu\nu\alpha\beta} - \mathrm{i}g\{c \overset{*}{,} \overline{Q}_{\mu\nu\alpha\beta}\})$$

$$-\mathrm{i}\overline{N}^{\mu\nu\alpha\beta} * g[c \overset{*}{,} J_{\mu\nu\alpha\beta}] - \mathrm{i}N^{\mu\nu\alpha\beta} * g[c \overset{*}{,} \overline{J}_{\mu\nu\alpha\beta}] \Bigg\}, \tag{14}$$

and it is ready for the BRST analysis.

3. Equations compatible with the quantum action principle

In this section, we will present several symmetries compatible with the QAP, which will be useful in the BRST renormalization procedure. First we have the traditional Ward identities present in usual gauge theories.

- Slavnov Taylor

$$S(\Sigma) = \int \mathrm{d}^4 x \Bigg\{ \frac{\delta\Sigma}{\delta A_\mu}\frac{\delta\Sigma}{\delta\Omega^\mu} + \frac{\delta\Sigma}{\delta c}\frac{\delta\Sigma}{\delta L} + \mathrm{i}b\frac{\delta\Sigma}{\delta\overline{c}} + \frac{\delta\Sigma}{\delta\overline{u}^{\mu\nu}}\frac{\delta\Sigma}{\delta\xi_{\mu\nu}} + \frac{\delta\Sigma}{\delta u^{\mu\nu}}\frac{\delta\Sigma}{\delta\overline{\xi}_{\mu\nu}}$$

$$+ \frac{\delta\Sigma}{\delta\overline{v}^{\mu\nu}}\frac{\delta\Sigma}{\delta B_{\mu\nu}} + \frac{\delta\Sigma}{\delta v^{\mu\nu}}\frac{\delta\Sigma}{\delta\overline{B}_{\mu\nu}} + \frac{\delta\Sigma}{\delta\overline{P}^{\mu\nu}}\frac{\delta\Sigma}{\delta\psi_{\mu\nu}} + \frac{\delta\Sigma}{\delta P^{\mu\nu}}\frac{\delta\Sigma}{\delta\overline{\psi}_{\mu\nu}}$$

$$+ \frac{\delta\Sigma}{\delta\overline{R}^{\mu\nu}}\frac{\delta\Sigma}{\delta\chi_{\mu\nu}} + \frac{\delta\Sigma}{\delta R^{\mu\nu}}\frac{\delta\Sigma}{\delta\overline{\chi}_{\mu\nu}} + \frac{\delta\Sigma}{\delta J^{\sigma\lambda\alpha\beta}}\frac{\delta\Sigma}{\delta\overline{N}_{\sigma\lambda\alpha\beta}} + \frac{\delta\Sigma}{\delta\overline{J}^{\sigma\lambda\alpha\beta}}\frac{\delta\Sigma}{\delta N_{\sigma\lambda\alpha\beta}}$$

$$+ \frac{\delta\Sigma}{\delta Q^{\sigma\lambda\alpha\beta}}\frac{\delta\Sigma}{\delta\overline{M}_{\sigma\lambda\alpha\beta}} + \frac{\delta\Sigma}{\delta\overline{Q}^{\sigma\lambda\alpha\beta}}\frac{\delta\Sigma}{\delta M_{\sigma\lambda\alpha\beta}} \Bigg\}, \tag{15}$$

- Lagrange multiplier and antighost equation

$$\frac{\delta\Sigma}{\delta b} = \mathrm{i}\partial^\mu A_{\mu\nu}, \qquad \partial_\mu \frac{\delta\Sigma}{\delta\Omega_\mu} + \frac{\delta\Sigma}{\delta\overline{c}} = 0, \tag{16}$$

- Ghost equation

$$\mathcal{G}\Sigma = \int \mathrm{d}^4 x \frac{\delta\Sigma}{\delta c} = 0, \tag{17}$$

- $SL(2, R)$ equation

$$D(\Sigma) = \int \mathrm{d}^4 x \left\{ c\frac{\delta\Sigma}{\delta\overline{c}} - \mathrm{i}\frac{\delta\Sigma}{\delta b}\frac{\delta\Sigma}{\delta L} \right\} = 0. \tag{18}$$

It is important to emphasize here that, due to the Moyal structure, the possible breaking terms are vanishing when integrated.

Now, due to the fact that all couplings are derivative in the noncommutative $U(1)$ theory, we also have integrated equations of motion:

$$\int \mathrm{d}^4 x \frac{\delta\Sigma}{\delta\overline{\chi}^{\mu\nu}} = \int \mathrm{d}^4 x \left(\frac{4}{3}\{\overline{J}_{\alpha\beta\sigma\lambda} \overset{*}{,} J^{\alpha\beta\sigma\lambda}\}\chi_{\mu\nu} + P_{\mu\nu} \right),$$

$$\int \mathrm{d}^4 x \frac{\delta\Sigma}{\delta\chi^{\mu\nu}} = \frac{4}{3} \int \mathrm{d}^4 x (\{\overline{J}_{\alpha\beta\sigma\lambda} \overset{*}{,} J^{\alpha\beta\sigma\lambda}\}\overline{\chi}_{\mu\nu}),$$

$$\int \mathrm{d}^4 x \frac{\delta\Sigma}{\delta\overline{\psi}^{\mu\nu}} = -\frac{4}{3} \int \mathrm{d}^4 x (\{\overline{J}_{\alpha\beta\sigma\lambda} \overset{*}{,} J^{\alpha\beta\sigma\lambda}\}\psi_{\mu\nu}), \tag{19}$$

$$\int \mathrm{d}^4 x \frac{\delta\Sigma}{\delta\psi^{\mu\nu}} = \int \mathrm{d}^4 x \left(\frac{4}{3}\{\overline{J}_{\alpha\beta\sigma\lambda} \overset{*}{,} J^{\alpha\beta\sigma\lambda}\}\overline{\psi}_{\mu\nu} + \overline{R}_{\mu\nu} \right),$$

$$\int \mathrm{d}^4 x \frac{\delta\Sigma}{\delta\overline{\xi}^{\mu\nu}} = 0.$$

These Ward identities will play a major role in the renormalizability study that we will conduct. Let us observe here that these symmetries are only present in the $U(1)$ case for the general $U(N)$ theory has nonderivative interactions. The absence of the Ward identities (19) is the main reason why we believe that in the non-Abelian noncommutative case, we need an alternative way of approaching the IR/UV problem. Now, let us go back to the $U(1)$ case.

The final symmetries that we will list are the identities associated with the BRST doublet structure $U^{(1)}$,

$$
U^{(1)}{}_{\sigma\lambda\mu\nu}(\Sigma) = \int \mathrm{d}^4x \left(\xi_{\sigma\lambda}\frac{\delta\Sigma}{\delta B_{\mu\nu}} + \overline{B}_{\mu\nu}\frac{\delta\Sigma}{\delta\overline{\xi}_{\sigma\lambda}} + \overline{\chi}_{\mu\nu}\frac{\delta\Sigma}{\delta\overline{\psi}_{\sigma\lambda}} + \psi_{\sigma\lambda}\frac{\delta\Sigma}{\delta\chi_{\mu\nu}} \right.
$$
$$
+ \overline{J}_{\mu\nu\alpha\beta}\frac{\delta\Sigma}{\delta\overline{Q}_{\sigma\lambda\alpha\beta}} + M_{\sigma\lambda\alpha\beta}\frac{\delta\Sigma}{\delta N_{\mu\nu\alpha\beta}} + J_{\sigma\lambda\alpha\beta}\frac{\delta\Sigma}{\delta Q_{\mu\nu\alpha\beta}} + \overline{M}_{\mu\nu\alpha\beta}\frac{\delta\Sigma}{\delta\overline{N}_{\sigma\lambda\alpha\beta}}
$$
$$
\left. + u_{\sigma\lambda}\frac{\delta\Sigma}{\delta v_{\mu\nu}} + \overline{v}_{\mu\nu}\frac{\delta\Sigma}{\delta\overline{u}_{\sigma\lambda}} + P_{\sigma\lambda}\frac{\delta\Sigma}{\delta R_{\mu\nu}} + \overline{R}_{\mu\nu}\frac{\delta\Sigma}{\delta\overline{P}_{\sigma\lambda}} \right) = 0, \tag{20}
$$

the linearly broken symmetries $U^{(0)}$ and $\widetilde{U}^{(0)}$,

$$
U^{(0)}{}_{\sigma\lambda\mu\nu}(\Sigma) = -\Theta^{(0)}{}_{\sigma\lambda\mu\nu}
$$
$$
U^{(0)}{}_{\sigma\lambda\mu\nu}(\Sigma) = \int \mathrm{d}^4x \left(B_{\sigma\lambda}\frac{\delta\Sigma}{\delta B_{\mu\nu}} - \overline{B}_{\mu\nu}\frac{\delta\Sigma}{\delta\overline{B}_{\sigma\lambda}} + \chi_{\sigma\lambda}\frac{\delta\Sigma}{\delta\chi_{\mu\nu}} - \overline{\chi}_{\mu\nu}\frac{\delta\Sigma}{\delta\overline{\chi}_{\sigma\lambda}} \right.
$$
$$
+ J_{\sigma\lambda\alpha\beta}\frac{\delta\Sigma}{\delta J_{\mu\nu\alpha\beta}} - \overline{N}_{\mu\nu\alpha\beta}\frac{\delta\Sigma}{\delta\overline{N}_{\sigma\lambda\alpha\beta}} - \overline{J}_{\mu\nu\alpha\beta}\frac{\delta\Sigma}{\overline{J}_{\sigma\lambda\alpha\beta}} + N_{\sigma\lambda\alpha\beta}\frac{\delta\Sigma}{N_{\mu\nu\alpha\beta}}
$$
$$
\left. + R_{\sigma\lambda}\frac{\delta\Sigma}{\delta R_{\mu\nu}} - \overline{R}_{\mu\nu}\frac{\delta\Sigma}{\delta\overline{R}_{\sigma\lambda}} + v_{\sigma\lambda}\frac{\delta\Sigma}{\delta v_{\mu\nu}} - \overline{v}_{\mu\nu}\frac{\delta\Sigma}{\delta\overline{v}_{\sigma\lambda}} \right)
$$
$$
\Theta^{(0)}{}_{\sigma\lambda\mu\nu} = \int \mathrm{d}^4x (u_{\sigma\lambda} * \overline{B}_{\mu\nu} + P_{\sigma\lambda} * \overline{\chi}_{\mu\nu} + \overline{R}_{\mu\nu} * \psi_{\sigma\lambda} + \overline{v}_{\mu\nu} * \xi_{\sigma\lambda}
$$
$$
+ \overline{J}_{\mu\nu}{}^{\alpha\beta} * M_{\sigma\lambda\alpha\beta} - \overline{M}_{\mu\nu}{}^{\alpha\beta} * J_{\sigma\lambda\alpha\beta}), \tag{21}
$$

$$
\widetilde{U}^{(0)}_{\sigma\lambda\mu\nu}(\Sigma) = \Theta^{(0)}{}_{\mu\nu\sigma\lambda}
$$
$$
\widetilde{U}^{(0)}_{\sigma\lambda\mu\nu}(\Sigma) = \int \mathrm{d}^4x \left(\psi_{\mu\nu}\frac{\delta\Sigma}{\delta\psi_{\sigma\lambda}} - \overline{\psi}_{\sigma\lambda}\frac{\delta\Sigma}{\delta\overline{\psi}_{\mu\nu}} + \xi_{\mu\nu}\frac{\delta\Sigma}{\delta\xi_{\sigma\lambda}} - \overline{\xi}_{\sigma\lambda}\frac{\delta\Sigma}{\delta\overline{\xi}_{\mu\nu}} \right.
$$
$$
- \overline{Q}_{\sigma\lambda\alpha\beta}\frac{\delta\Sigma}{\delta\overline{Q}_{\mu\nu\alpha\beta}} + M_{\mu\nu\alpha\beta}\frac{\delta\Sigma}{\delta M_{\sigma\lambda\alpha\beta}} + Q_{\mu\nu\alpha\beta}\frac{\delta\Sigma}{\delta Q_{\sigma\lambda\alpha\beta}} - \overline{M}_{\sigma\lambda\alpha\beta}\frac{\delta\Sigma}{\delta\overline{M}_{\mu\nu\alpha\beta}}
$$
$$
\left. + P_{\mu\nu}\frac{\delta\Sigma}{\delta P_{\sigma\lambda}} - \overline{P}_{\sigma\lambda}\frac{\delta\Sigma}{\delta\overline{P}_{\mu\nu}} + u_{\mu\nu}\frac{\delta\Sigma}{\delta u_{\sigma\lambda}} - \overline{u}_{\sigma\lambda}\frac{\delta\Sigma}{\delta\overline{u}_{\mu\nu}} \right), \tag{22}
$$

which together define the reality constraint on our action Σ and an associated reality charge Q for all the fields and sources of the theory,

$$
Q = \mathrm{Tr}\, U^{(0)} + \mathrm{Tr}\, \widetilde{U}^{(0)}, \tag{23}
$$

and finally the last two symmetries

$$
U^{(2)}{}_{\sigma\lambda\mu\nu}(\Sigma) = \int \mathrm{d}^4x \left\{ \psi_{\sigma\lambda}\frac{\delta\Sigma}{\delta\overline{\xi}^{\mu\nu}} + \psi_{\mu\nu}\frac{\delta\Sigma}{\delta\overline{\xi}^{\sigma\lambda}} - u_{\sigma\lambda}\frac{\delta\Sigma}{\delta\overline{P}^{\mu\nu}} - u_{\mu\nu}\frac{\delta\Sigma}{\delta\overline{P}^{\sigma\lambda}} \right\} = 0, \tag{24}
$$

Table 1. Quantum numbers of the fields.

Fields	A	b	c	$\bar{c}$	ψ	$\bar{\psi}$	ξ	$\bar{\xi}$	χ	$\bar{\chi}$	B	$\bar{B}$
UV dimension	1	2	0	2	1	1	1	1	1	1	1	1
Ghost number	0	0	1	−1	1	−1	1	−1	0	0	0	0
Q charge	0	0	0	0	1	−1	1	−1	1	−1	1	−1
Statistics	co	co	an	an	an	an	an	an	co	co	co	co

Table 2. Quantum numbers of the sources.

Sources	Ω	L	J	$\bar{J}$	Q	$\bar{Q}$	u	$\bar{u}$	v	$\bar{v}$	P	$\bar{P}$	R	$\bar{R}$
UV dimension	3	4	1	1	1	1	3	3	3	3	3	3	3	3
Ghost number	−1	−2	0	0	−1	−1	0	−2	−1	−1	0	−2	−1	−1
Q charge	0	0	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1
Statistics	an	co	co	co	an	an	co	co	an	an	co	co	an	an

Table 3. Quantum numbers of the auxiliary sources.

Sources	M	N	$\bar{M}$	$\bar{N}$
UV dimension	3	3	3	3
Ghost number	0	−1	0	−1
Q charge	1	1	−1	−1
Statistics	co	an	co	an

and

$$\widetilde{U}^{(2)}_{\sigma\lambda\mu\nu}(\Sigma) = \int \mathrm{d}^4x \left\{ \psi_{\sigma\lambda} \frac{\delta\Sigma}{\delta\bar{\psi}^{\mu\nu}} + \psi_{\mu\nu} \frac{\delta\Sigma}{\delta\bar{\psi}^{\sigma\lambda}} + \xi_{\sigma\lambda} \frac{\delta\Sigma}{\delta\bar{\xi}^{\mu\nu}} + \xi_{\mu\nu} \frac{\delta\Sigma}{\delta\bar{\xi}^{\sigma\lambda}} \right.$$
$$\left. - u_{\sigma\lambda} \frac{\delta\Sigma}{\delta\bar{u}^{\mu\nu}} - u_{\mu\nu} \frac{\delta\Sigma}{\delta\bar{u}^{\sigma\lambda}} - P_{\sigma\lambda} \frac{\delta\Sigma}{\delta\bar{P}^{\mu\nu}} - P_{\mu\nu} \frac{\delta\Sigma}{\delta\bar{P}^{\sigma\lambda}} \right\} = 0. \tag{25}$$

Let us explain here that the tensorial nature of these symmetries will be responsible for the fact that, in the cohomological analysis that we will undertake, the only possible Lorentz indices contractions of the fields $\chi, \bar{\chi}, B, \bar{B}, \psi, \bar{\psi}, \xi, \bar{\xi}$ and their sources obey the same structure present in the action (14).

4. Stability of the quantum action

In order to study the stability of the quantum action, let us start by presenting the quantum numbers of all fields and sources (see tables 1–3).

Once more, we call attention to the fact that in the stability analysis of the quantum action it is necessary to take into account not only the canonical dimension but also the ultraviolet dimension of all fields. The use of canonical dimensions generally leads to an incorrect cohomological analysis.

4.1. The invariant counterterm

In this section we will focus our attention on the possible UV counterterms that can change the propagation behavior of the classical theory. The original structure that is obtained from (14) when the sources attain their physical values is specially designed in order to incorporate the coefficient of the IR singularity appearing in the two-point function of the noncommutative $U(1)$ theory [17] (other singular IR contributions are not addressed in this analysis [29]). Then, new UV counterterms different from those already present in the starting action (14) can be rather harmful to the delicate match at the IR level. The search for such contributions is our main interest here.

Before proceeding, we would like to mention the use of the QAP in this noncommutative context. Let us recapitulate the origin of the IR/UV mix. In general, Feynman graph calculations in noncommutative theories can be divided in planar and non-planar contributions [12]. The latter are those characterized by the presence of a remaining phase inside the Feynman integrals. This phase is responsible for the damping of the UV divergences, which become naturally regularized. As this phase depends on the external momenta (the phase disappears for vanishing external momenta), the would-be UV divergence is turned into an UV finite but IR singular contribution. The introduction of these nonlocal objects in the starting action is a possible mechanism that is actually behind the reasoning leading to the proposal of the action (1) to account for the two-point function IR singularity of the pure noncommutative $U(1)$ theory. On the other side, the non-planar graph is accompanied by its planar counterpart, when the phase becomes dependent only on the external momenta, and, in this way, factorizes off the integral. In general, the UV divergence of a planar graph is accompanied by the non-planar singularity, generating the IR/UV mix. But the point is that the $U(1)$ planar graphs, where the UV divergences are generated, mimic the structure of a commutative theory inside the integrals, with phase-dependent coefficients restoring the noncommutative vertices [19]. This is what we meant by the Moyal rigidity hypothesis in the introduction. Then, it is in this sector of the noncommutative theory that the QAP seems to be valid, with a power-counting bounding the possible UV counterterms. The use of the QAP can then be a guidance to search for possible IR singularities in the non-planar counterparts associated with the planar UV divergent contributions. Finally, once all IR singularities of a previous theory are stabilized, using mechanisms as that in (1), the question if the new nonlocal action develops new IR singularities can again be answered using the QAP in the planar sector. If the QAP indicates that this IR stabilized theory has no new UV divergent contributions, we would be meeting a renormalization condition for this final theory. In order to characterize any invariant counterterm which can be added freely to all orders in perturbation theory [18], we perturb the classical action Σ by adding an arbitrary integrated local polynomial Σ^{count} of dimension up-bounded by four, vanishing ghost number and Q charge. We demand that $\Gamma = \Sigma + \epsilon \Sigma^{\text{count}} + O(\epsilon^2)$, where ϵ is a small expansion parameter, satisfies the same Ward identities as Σ. This requirement provides the following constraints on the counterterm (for convenience of the reader, we follow the same sequence of Ward identities of section (3)):

$$\mathcal{B}_\Sigma \Sigma^{\text{count}} = 0, \tag{26}$$

$$\frac{\delta}{\delta b} \Sigma^{\text{count}} = 0, \tag{27}$$

$$\partial_\mu \frac{\delta \Sigma^{\text{count}}}{\delta \Omega_\mu} + \frac{\delta \Sigma^{\text{count}}}{\delta \overline{c}} = 0, \tag{28}$$

$$D_\Sigma \Sigma^{\text{count}} = 0, \tag{29}$$

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$$\mathcal{G}\Sigma^{\text{count}} = 0, \tag{30}$$

$$\int \mathrm{d}^4x \, \frac{\delta\Sigma^{\text{count}}}{\delta\overline{\chi}^{\mu\nu}} = 0, \tag{31}$$

$$\int \mathrm{d}^4x \, \frac{\delta\Sigma^{\text{count}}}{\delta\chi^{\mu\nu}} = 0, \tag{32}$$

$$\int \mathrm{d}^4x \, \frac{\delta\Sigma^{\text{count}}}{\delta\overline{\psi}^{\mu\nu}} = 0, \tag{33}$$

$$\int \mathrm{d}^4x \, \frac{\delta\Sigma^{\text{count}}}{\delta\psi^{\mu\nu}} = 0, \tag{34}$$

$$\int \mathrm{d}^4x \, \frac{\delta\Sigma^{\text{count}}}{\delta\overline{\xi}^{\mu\nu}} = 0, \tag{35}$$

$$U^{(1)}{}_{\sigma\lambda\mu\nu}(\Sigma^{\text{count}}) = 0, \tag{36}$$

$$U^{(0)}{}_{\sigma\lambda\mu\nu}(\Sigma^{\text{count}}) = 0, \tag{37}$$

$$\widetilde{U}^{(0)}_{\sigma\lambda\mu\nu}(\Sigma^{\text{count}}) = 0, \tag{38}$$

$$U^{(2)}{}_{\sigma\lambda\mu\nu}(\Sigma^{\text{count}}) = 0, \tag{39}$$

$$\widetilde{U}^{(2)}_{\sigma\lambda\mu\nu}(\Sigma^{\text{count}}) = 0, \tag{40}$$

where in (26), $\mathcal{B}_\Sigma$ stands for the nilpotent linearized Slavnov–Taylor operator,

$$
\begin{aligned}
\mathcal{B}_\Sigma = \int \mathrm{d}^4x \Bigg(& \frac{\delta\Sigma}{\delta A_\mu}\frac{\delta}{\delta\Omega^\mu} + \frac{\delta\Sigma}{\delta\Omega^\mu}\frac{\delta}{\delta A_\mu} + \frac{\delta\Sigma}{\delta c}\frac{\delta}{\delta L} + \frac{\delta\Sigma}{\delta L}\frac{\delta}{\delta c} + \mathrm{i}b\frac{\delta}{\delta\overline{c}} \\
& + \frac{\delta\Sigma}{\delta\overline{u}^{\mu\nu}}\frac{\delta}{\delta\xi_{\mu\nu}} + \frac{\delta\Sigma}{\delta\xi_{\mu\nu}}\frac{\delta}{\delta\overline{u}^{\mu\nu}} + \frac{\delta\Sigma}{\delta u^{\mu\nu}}\frac{\delta}{\delta\overline{\xi}_{\mu\nu}} + \frac{\delta\Sigma}{\delta\overline{\xi}_{\mu\nu}}\frac{\delta}{\delta u^{\mu\nu}} \\
& + \frac{\delta\Sigma}{\delta\overline{v}^{\mu\nu}}\frac{\delta}{\delta B_{\mu\nu}} + \frac{\delta\Sigma}{\delta B_{\mu\nu}}\frac{\delta}{\delta\overline{v}^{\mu\nu}} + \frac{\delta\Sigma}{\delta v^{\mu\nu}}\frac{\delta}{\delta\overline{B}_{\mu\nu}} + \frac{\delta\Sigma}{\delta\overline{B}_{\mu\nu}}\frac{\delta}{\delta v^{\mu\nu}} \\
& + \frac{\delta\Sigma}{\delta\overline{P}^{\mu\nu}}\frac{\delta}{\delta\psi_{\mu\nu}} + \frac{\delta\Sigma}{\delta\psi_{\mu\nu}}\frac{\delta}{\delta\overline{P}^{\mu\nu}} + \frac{\delta\Sigma}{\delta P^{\mu\nu}}\frac{\delta}{\delta\overline{\psi}_{\mu\nu}} + \frac{\delta\Sigma}{\delta\overline{\psi}_{\mu\nu}}\frac{\delta}{\delta P^{\mu\nu}} \\
& + \frac{\delta\Sigma}{\delta\overline{R}^{\mu\nu}}\frac{\delta}{\delta\chi_{\mu\nu}} + \frac{\delta\Sigma}{\delta\chi_{\mu\nu}}\frac{\delta}{\delta\overline{R}^{\mu\nu}} + \frac{\delta\Sigma}{\delta R^{\mu\nu}}\frac{\delta}{\delta\overline{\chi}_{\mu\nu}} + \frac{\delta\Sigma}{\delta\overline{\chi}_{\mu\nu}}\frac{\delta}{\delta R^{\mu\nu}} \\
& + \frac{\delta\Sigma}{\delta J_{\sigma\lambda\alpha\beta}}\frac{\delta}{\delta\overline{N}^{\sigma\lambda\alpha\beta}} + \frac{\delta\Sigma}{\delta N^{\sigma\lambda\alpha\beta}}\frac{\delta}{\delta\overline{J}_{\sigma\lambda\alpha\beta}} + \frac{\delta\Sigma}{\delta\overline{J}_{\sigma\lambda\alpha\beta}}\frac{\delta}{\delta N^{\sigma\lambda\alpha\beta}} + \frac{\delta\Sigma}{\delta\overline{N}^{\sigma\lambda\alpha\beta}}\frac{\delta}{\delta J_{\sigma\lambda\alpha\beta}} \\
& + \frac{\delta\Sigma}{\delta Q_{\sigma\lambda\alpha\beta}}\frac{\delta}{\delta\overline{M}^{\sigma\lambda\alpha\beta}} + \frac{\delta\Sigma}{\delta M^{\sigma\lambda\alpha\beta}}\frac{\delta}{\delta\overline{Q}_{\sigma\lambda\alpha\beta}} + \frac{\delta\Sigma}{\delta\overline{Q}_{\sigma\lambda\alpha\beta}}\frac{\delta}{\delta M^{\sigma\lambda\alpha\beta}} + \frac{\delta\Sigma}{\delta\overline{M}^{\sigma\lambda\alpha\beta}}\frac{\delta}{\delta Q_{\sigma\lambda\alpha\beta}} \Bigg).
\end{aligned}
$$

$$\mathcal{B}_\Sigma^2 = 0, \tag{41}$$

and in (29),

$$D_\Sigma = \int \mathrm{d}^4x \left\{ c\frac{\delta}{\delta\overline{c}} - \mathrm{i}\frac{\delta\Sigma}{\delta b}\frac{\delta}{\delta L} - \mathrm{i}\frac{\delta\Sigma}{\delta L}\frac{\delta}{\delta b} \right\} = 0. \tag{42}$$

The first constraint (26), together with (41), establishes a cohomological problem for the operator $\mathcal{B}_\Sigma$ and its solution is given by [18]:

$$\Sigma^{\text{count}} = \frac{a_0}{4}\int \mathrm{d}^4x \, F_{\mu\nu} * F^{\mu\nu} + \Delta^{(0)}, \qquad \Delta^{(0)} = \mathcal{B}_\Sigma \Delta^{(-1)}, \tag{43}$$

where $\Delta^{(0)}$ is a local integrated polynomial in all fields and sources, with ultraviolet dimension up-bounded by four, ghost number zero and vanishing Q charge. The other Ward identities (27)–(40) will give constraints to $\Delta^{(0)}$. In the first place, equations (27) and (28) state that b cannot be used in its construction, and that the source Ω_μ and the antighost $\overline{c}$ can only appear in the combination $\Omega_\mu + \partial_\mu \overline{c}$. Equations (29) and (30) are also typical of gauge theories and fix coefficients of counterterms already present at the original action. Now, it is of fundamental importance to note that, due to equations (31)–(35), the fields $\overline{\chi}, \chi, \overline{\psi}, \psi$ and $\overline{\xi}$ only appear directly derivated or inside Moyal commutators (anticommutators). In fact, this is also valid for all BRST sources in the theory, which obey similar equations.

Now, if we concentrate on contributions that can damage the IR equilibrium established in (1), we must look for counterterms that may modify the gauge propagation coming from this action. The first one that comes to mind is

$$\int \mathrm{d}^4 x (\overline{B}_{\mu\nu} B^{\mu\nu} - \overline{\xi}_{\mu\nu} \xi^{\mu\nu}), \tag{44}$$

which, although being allowed by all the remaining Ward identities, is avoided by equation (35).

Another possible counterterm which deserves special attention is

$$\int \mathrm{d}^4 x (\overline{B}_{\mu\nu} D^2 B^{\mu\nu} - \overline{\xi}_{\mu\nu} D^2 \xi^{\mu\nu}), \tag{45}$$

which is not allowed explicitly by the identity (39).

There is also the element

$$a \int \mathrm{d}^4 x (\overline{\chi}_{\mu\nu} D^2 \chi^{\mu\nu} - \overline{\psi}_{\mu\nu} D^2 \psi^{\mu\nu}), \tag{46}$$

which is in fact allowed by all the symmetries. This counterterm, not originally present in the localized action (14), changes the propagator to

$$\langle A(k)_\mu A_\nu(-k) \rangle = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \frac{k^2}{k^4 + a\gamma^2 k^2 + \gamma^4}. \tag{47}$$

This form of the propagator still means that the IR ambiguity is eliminated, as one can see by rewriting the new nonlocal theory in the presence of the term (46).

Now, we should not forget that the original theory, equation (5), that we are studying is actually a limit of the larger theory described by (14), when the sources reach their physical values (10). Then, there is still a class of possible counterterms that eventually can change the propagator (or the nonlocal action) but that appear in the larger theory as four-point divergent contributions. In particular, we have that the element

$$\alpha \int \mathrm{d}^4 x (\{\overline{J}_{\alpha\beta\sigma\lambda} \overset{*}{,} J^{\alpha\beta\sigma\lambda}\}\{\overline{B}_{\mu\nu} \overset{*}{,} B^{\mu\nu}\} - \{\overline{Q}_{\alpha\beta\sigma\lambda} \overset{*}{,} J^{\alpha\beta\sigma\lambda}\}\{\overline{B}_{\mu\nu} \overset{*}{,} \xi^{\mu\nu}\}) \tag{48}$$

is also allowed by all the symmetries from (26)–(40).

These two terms are then responsible for a gauge propagator modified in relation to that in (3). When the sources $J, \overline{J}, Q$ and $\overline{Q}$ are set to their physical values, the propagator for the gauge field takes the general form

$$\langle A(k)_\mu A_\nu(-k) \rangle = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \frac{\Xi^2}{k^2(\Xi^2 + \gamma^2 \Pi)}$$
$$\Xi = k^4 - \alpha a \gamma^2 k^2 + \alpha \gamma^4 \tag{49}$$
$$\Pi = ak^6 + (1 - \alpha a^2)\gamma^2 k^4 - \alpha \gamma^6.$$

This means that the inclusion of all counterterms in the starting classical action will in the end destroy the IR mechanism proposed in (1). Unfortunately, the element (48) seems to be found in explicit graphic calculations and it is clear that only the case $\alpha = 0$ would correspond to a well-behaved propagator.

It is important to mention that the cohomological analysis extended to the noncommutative space is constrained by the Ward identities of the action. If another Ward identity is observed, this constraint may reduce the number of counterterms. One example is the counterterm responsible for the mass a that is apparently not required in one-loop calculations.

It should be stressed that although the choice of a and α different from 0 at tree level would give rise to a very different type of propagator, the ultraviolet behavior is exactly $\frac{1}{k^2}$. With an adequate choice for these parameters, it is possible that the propagator satisfies the Wilson criterion for confinement [30, 31]. The Wilson criterion and the loss of positivity are interpreted as a sign of confinement [32–35]. This would possibly mean that confining phases can be expected in noncommutative gauge theories. In such a case, the physical excitations are not associated with the fundamental fields and only condensates of fields are good candidates to physical states of the model [35]. Another important point is that in this context the Wick rotation is not allowed in general. But there is still the possibility that the correlators between two condensates have a massive particle pole. These correlators admit Wick rotation and can be associated with observable physical states in Minkowsky space [35]. These observations may be useful for a future understanding of the nature of noncommutative gauge theories.

5. Conclusion

We saw during this work how a nonlocal mechanism as that in equation (1), that can classically cure the infrared problem of the two-point function of a noncommutative Maxwell theory, is not ultraviolet stable.

In the development of this algebraic proof, we followed the approach used by [32], and more recently improved by Sorella and Baulieu [28], to the study of the BRST quantization of the nonlocal action coming from Gribov's observations on the infrared properties of gauge theories. We understand that, if in the usual commutative space the use of nonlocal actions is an alternative option to the study of the infrared regime, on the other hand, in the noncommutative case this seems to be the inevitable path to solve the intrinsic problem of the IR/UV mix.

As a final comment, we would like to point out a recent proposal simplifying (1) in order to avoid the quantum generation of counterterms as (48), but still preserving the IR match for the two-point function [36].

Acknowledgments

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